Subject. Integral (L.) Date. 14 Feb 20

Definite Integral.

Riemann Sums.

Remember that :-

$$\sum_{K=1}^{n} a_K = a_1 + a_2 + \cdots + a_n$$

$$\sum_{K=1}^{n} a = a \sum_{K=1}^{n} K^{0} = n\,a$$

$$\begin{aligned}\sum_{K=1}^{n} (a_K - a_{K-1}) &= a_1 - a_0 \\ &+ \cancel{a_2} - \cancel{a_1} \\ &+ a_3 - \cancel{a_2} \\ &+ a_n - \cancel{a_{n-1}}\end{aligned} = a_n - a_0$$

$$\sum_{K=1}^{n} K = \frac{n}{2}(n+1)$$ عددية :- جـن = ن/٢ (أ + ل)

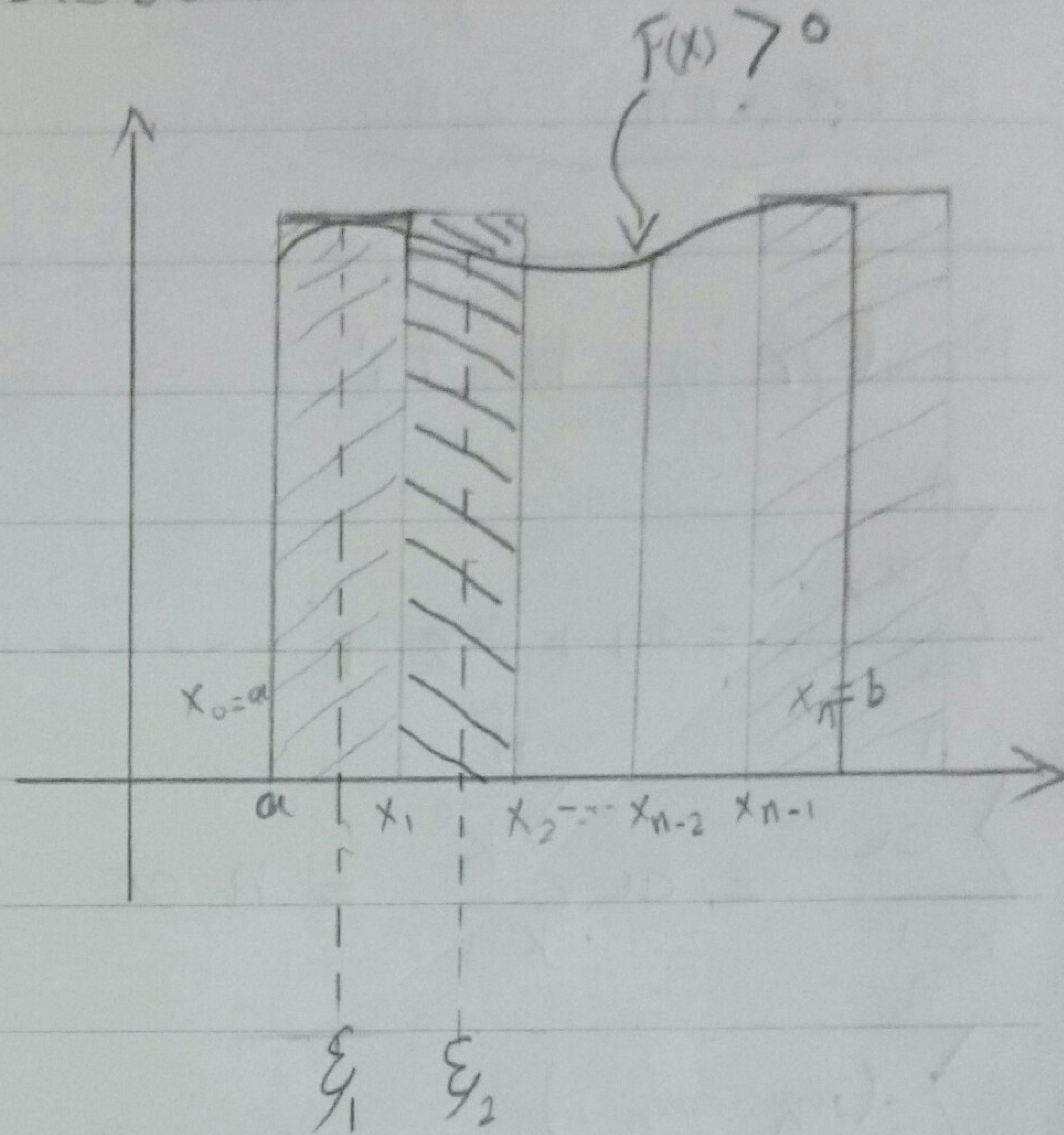
$$\sum_{K=1}^{n} a^K = \frac{a(1-a^n)}{1-a}$$ هندسية :-

Some Series :-

$$\sum_{1}^{n} K^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$$

Bernahrd Riemann 1826-1866 :-



$$A = F(\xi_1)(x_1 - x_0) + F(\xi_2)(x_2 - x_1)$$

$$+ \cdots\cdots + F(\xi_n)(X_n - X_{n-1})$$

$$A \simeq \sum_{k=1}^{n} F(\xi_k)\,\Delta X_k$$

$$\Delta X_k = X_k - X_{k-1}$$

$$\Delta X_k = \frac{b-a}{n}$$

$$X_1 = X_0 + \frac{b-a}{n}$$

$$X_2 = X_1 + \frac{b-a}{n} = X_0 + 2\,\frac{b-a}{n}$$

$$\vdots$$

$$X_n = a + \frac{b-a}{n}\,k$$

$\xi_1 = X_1$

$\xi_2 = X_2$

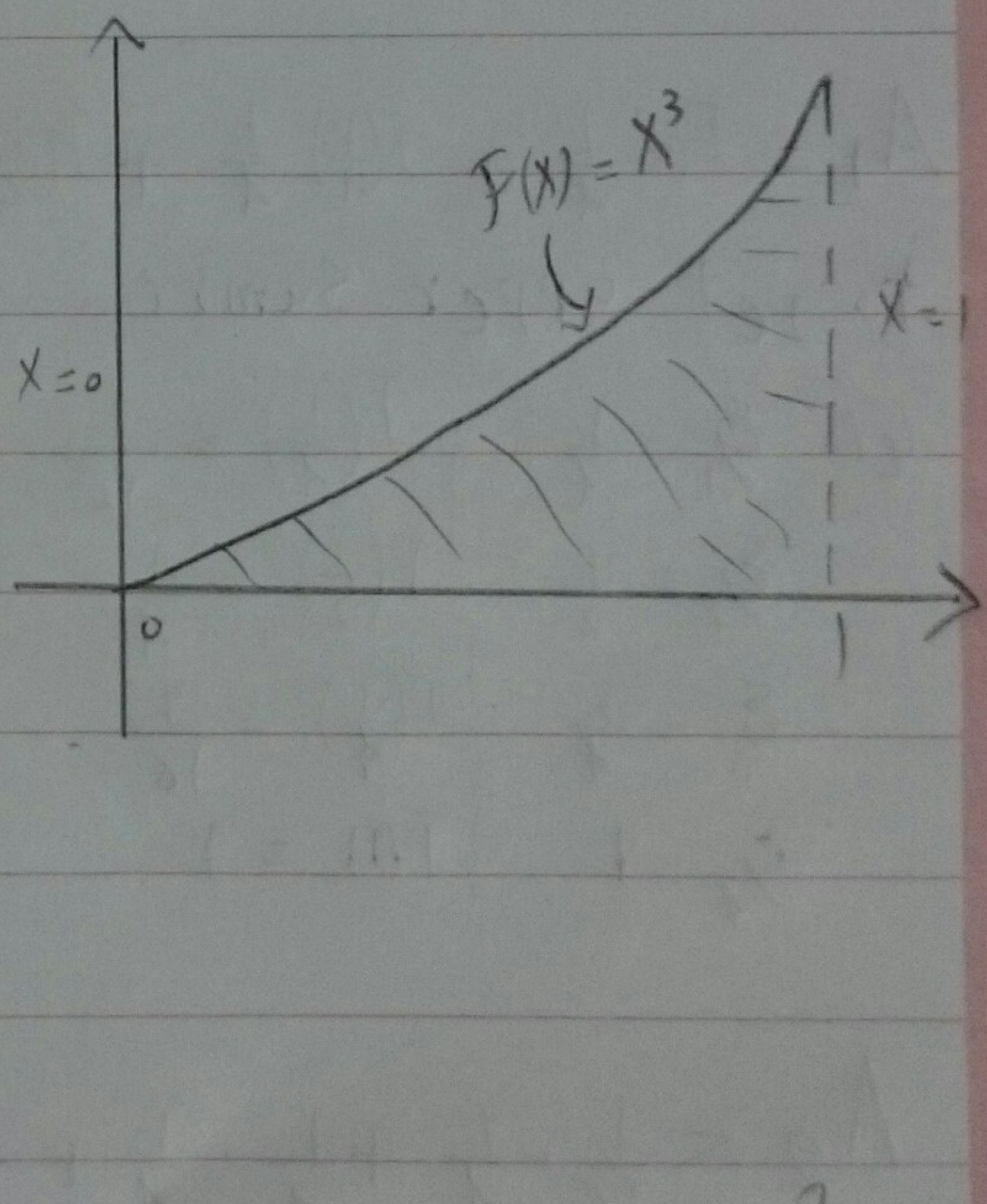
$$A \simeq \sum_{k=1}^{n} F(X_k)\left(\frac{b-a}{n}\right)$$

$$= \sum_{k=1}^{n} F\left(a + \frac{b-a}{n}k\right)\left(\frac{b-a}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{b-a}{n}\right) \cdot F\left(a + \frac{b-a}{n}k\right)$$

EX①: Find the area bounded by $F(X) = X^3$, $X = 0$, $y = 0$ and $X = 1$ using Riemann Sums.

Sol:



$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1-0}{n}\right)\left(0 + \frac{1-0}{n}k\right)^3$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k^3}{n^3} = \lim \frac{1}{n^4} \sum_{k=1}^{n} k^3$$

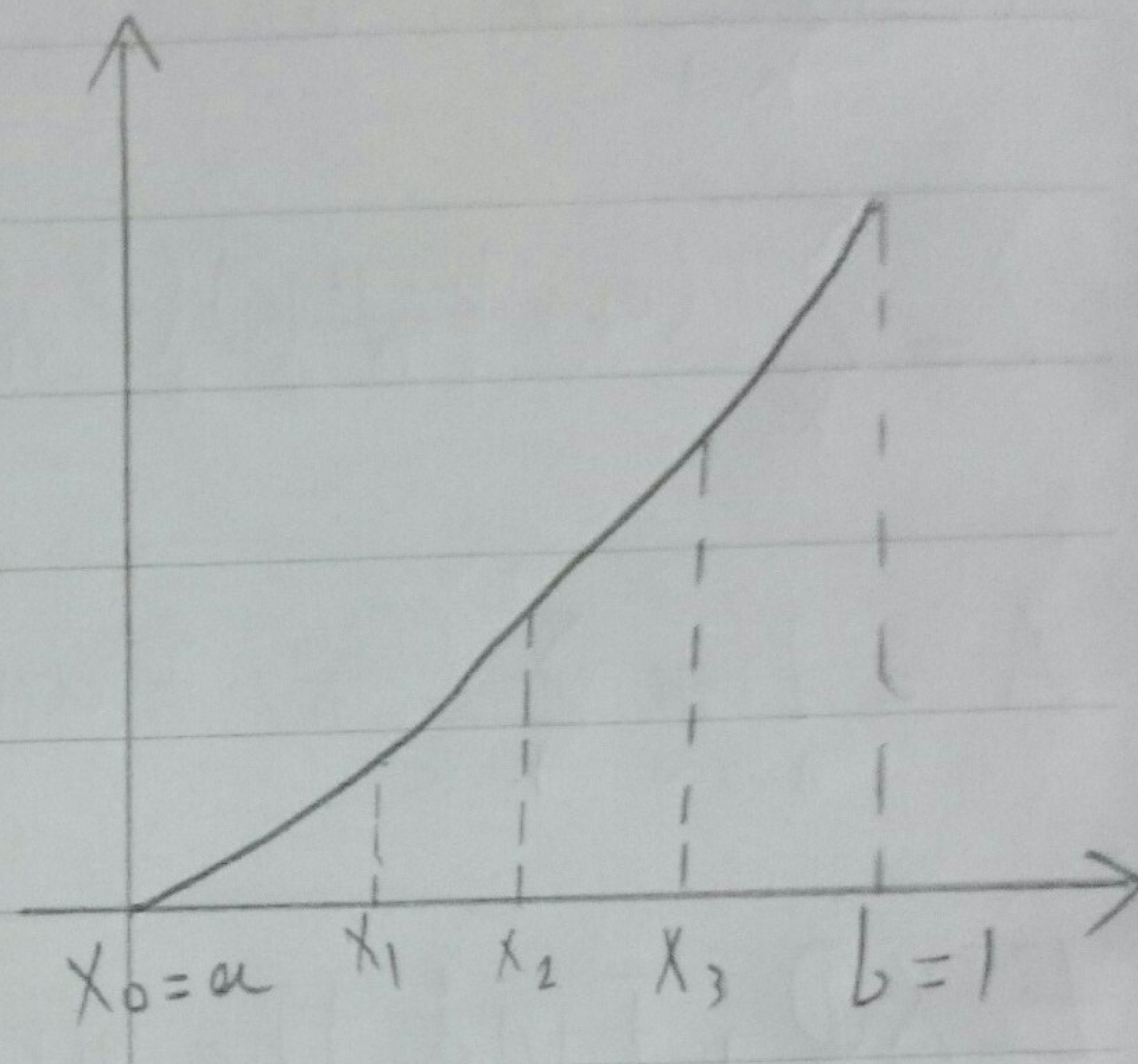
$$\sum_{1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$A = \lim_{n\to\infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} = \lim_{n\to\infty} \frac{1}{4}\, \frac{n^2}{n^2}\, \frac{(n+1)^2}{n^2} = \lim \frac{1}{4} \cdot 1 \cdot \left(1 + \frac{1}{n}\right)^2$$

$$= \boxed{\frac{1}{4}}$$

EX ③: Find the upper and Lower Sums of $F(x) = x^2$ on the Closed interval $[0,1]$ which correspond the partition:-

$P_n = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$



Sol.

to get Lower Area

let
$\xi_1 = 0$ | $F(0) = 0$
$\xi_2 = \frac{1}{4}$ | $F(\frac{1}{4}) = \frac{1}{16}$
$\xi_3 = \frac{1}{2}$ | $F(\frac{1}{2}) = \frac{1}{4}$
$\xi_4 = \frac{3}{4}$ | $F(\frac{3}{4}) = \frac{9}{16}$

$$A_1 = F(0)\frac{1}{4} + F(\tfrac{1}{4})\frac{1}{4} + F(\tfrac{1}{2})\frac{1}{4} + F(\tfrac{3}{4})\frac{1}{4} = \frac{7}{32} \#$$

to get upper Sums:-

let
$\xi_1 = \frac{1}{4}$ | $F(\frac{1}{4}) = \frac{1}{16}$
$\xi_2 = \frac{1}{2}$ | $F(\frac{1}{2}) = \frac{1}{4}$
$\xi_3 = \frac{3}{4}$ | $F(\frac{3}{4}) = \frac{9}{16}$
$\xi_4 = 1$ | $F(1) = 1$

$$A_2 = \frac{1}{16}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} + \frac{9}{16}\cdot\frac{1}{4} + 1\cdot\frac{1}{4} = \frac{15}{32} \#$$

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to you:- Evaluate $\int_1^3 3x^2 \, dx$

by using Riemann Sums.